# ABSORCIÓN DE VIBRACIONES EN EL CONTROL DE VELOCIDAD DE MOTORES DC

# HARMONIC OSCILLATION ABSORPTION IN VELOCITY CONTROL OF DC MOTORS

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**Resumen.** Se propone un enfoque de control dinámico para la regulación eficiente y robusta de la velocidad de un motor DC de imanes permanentes, sometido a torque de carga mecánico armónico. La síntesis del controlador de velocidad considera la inclusión de un absorbedor de vibración dinámico virtual, para suprimir activamente el torque de perturbación oscilatorio, generado por algún sistema mecánico usando un motor DC como actuador de movimiento. El absorbedor de vibración dinámico virtual se sintoniza a la frecuencia de excitación del torque de carga de perturbación a ser suprimido activamente. La propiedad estructural de planitud diferencial del sistema dinámico perturbado se usa para el diseño del controlador. La efectividad del absorbedor de vibración virtual embebido en el algoritmo de control por planitud diferencial se describe a través de resultados de simulación computacional.

Palabras clave: Absorbedor de vibración, motor dc, torque armónico, vibración síncrona.

**Abstract**. This paper introduces a dynamic control approach for efficient and robust regulation of velocity specified for the operation of magnet-permanent DC motors subjected to harmonic mechanical load torque. Synthesis of the velocity controller considers the inclusion of a virtual dynamic vibration absorber to actively suppress oscillating disturbance torque possibly generated by some mechanical load system using an electric DC motor as motion actuator. Nonphysical dynamic vibration absorber is thus tuned at the excitation frequency of the disturbance load torque to be actively suppressed. The structural property of differential flatness of the harmonically disturbed dynamic system is also used for the control design. Effectiveness of the virtual vibration absorber embedded into differential flatness control algorithm is shown by some computer simulation results.

Key Words: Vibration absorber, dc motor, harmonic torque, synchronous vibration.

# 1. Introduction

A dynamic vibration absorber is a control device used to attenuate undesirable vibrations around a specific excitation frequency. Diverse applications of physical vibration absorbers have shown their effectiveness into a certain attenuation band [1]. Effective vibration suppression of an unbalanced rotor system using vibration absorbers has been described in [2]. Vibration absorbers have been also applied to suppress vibration absorbers to attenuate harmonic oscillations in DC electric motors. This is, different to most contribution on vibration absorbers in mechanical engineering, unphysical vibration absorbers are synthesized from a dynamic control perspective. The present contribution provides important insights about how virtual vibration absorbers can be extended to suppress harmonic oscillating components in a wide class of controllable energy conversion systems. In fact, suppression of harmonic oscillations in modern power systems constitutes a relevant challenging issue [4-7]. Virtual vibration absorbers can be designed and tuned to suppress specified significant harmonics of voltage and current waveforms. A dynamic velocity control method based on virtual vibrations absorbers for magnet-permanent DC motors subjected to harmonic mechanical oscillations is proposed in this paper.

unbalance [8,9]. Nonphysical dynamic vibration absorber is thus tuned at the excitation frequency of the disturbance load torque to be actively suppressed. Then, efficient and robust regulation of rotor velocity is achieved. The structural property of differential flatness of the harmonically disturbed dynamic system is also exploited for control design. Thus, system variables can be described by formulas in terms of the so-called flat output variable and a finite number of its time derivatives [10]. Effectiveness of the virtual vibration absorber embedded into differential flatness control algorithm is shown by some computer simulation results. Hence, the proposed dynamic vibration absorption control approach represents a very good alternative for both suppression of synchronous harmonic vibrations and velocity regulation tasks in DC motors.

#### 2. Mathematical Model

Consider the dynamics of a DC permanent magnet motor with gearhead subjected to harmonic load torque  $\tau_L(t)$  [11, 12]

$$L\dot{I} = -RI - k_e n\omega + u$$

$$(J_1 + n^2 J_0)\dot{\omega} = -(b_1 + n^2 b_0)\omega + nk_m I - \tau_L$$
(1)

Here,  $\omega$  is the output angular velocity to be actively controlled in presence of harmonically disturbing torque, *I* is the electric current signal and *u* the voltage control input. The oscillating mechanical load torque is described as

$$\tau = \overline{\tau}_L \sin(\Omega t) , \qquad (2)$$

where  $\overline{\tau}_L$  is the amplitude and  $\Omega$  the angular frequency. Inductance and resistance of the armature circuit are respectively denoted by *L* and *R*.  $k_m$  represents the motor torque constant,  $k_e$  the back electromotive force constant, and *n* the speed reduction ratio of the gearhead.  $J_0$  and  $J_1$  are inertia moments of the motor and gearhead rotors.  $b_0$  and  $b_1$  are viscous damping constants of the motor and gearhead.

The differential flatness property is admitted by the DC motor. The flat output is given by the angular velocity  $y = \omega$ . Hence, differential parametrization of system variables in terms of the flat output and a finite number of its time derivatives is given by

$$\omega = y$$

$$I = \frac{J_e}{nk_m} \dot{y} + \frac{b_e}{nk_m} y + \frac{1}{nk_m} \tau_L$$

$$(3)$$

$$u = \frac{LJ_e}{nk_m} \ddot{y} + \left(\frac{b_e L + RJ_e}{nk_m}\right) \dot{y} + \left(\frac{n^2 k_e k_m + Rb_e}{nk_m}\right) y + \frac{L}{nk_m} \dot{\tau}_L + \frac{R}{nk_m} \tau_L$$

Thus, the angular velocity y is governed by the differential equation

$$\ddot{y} + a_1 \dot{y} + a_0 y = bu - \frac{R}{J_e L} \tau_L - \frac{1}{J_e} \dot{\tau}_L$$
(4)

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with

$$a_{0} = \frac{n^{2}k_{m}k_{e} + Rb_{e}}{J_{e}L}, \quad a_{1} = \frac{b_{e}}{J_{e}} + \frac{R}{L}, \quad b = \frac{nk_{m}}{J_{e}L}$$
(5)

where  $J_e$  is the total inertia moment and  $b_e$  is the equivalent damping constant.

## 3. Velocity Control with a Virtual Dynamic Vibration Absorber

Consider the angular velocity dynamics (4) rewritten as

$$\ddot{\mathbf{y}} + a_1 \dot{\mathbf{y}} + a_0 \mathbf{y} = b\mathbf{u} + F \tag{6}$$

where F stands for harmonic disturbances as well, which are described as

$$F = -\frac{R}{J_{eq}L}\tau_L - \frac{1}{J_{eq}}\dot{\tau}_L = \tau_L\sin(\Omega t + \phi)$$
(7)

Firstly, consider the undisturbed operational situation, i.e,  $F \equiv 0$ , the differential flatness-based velocity controller

$$u = \frac{1}{b} \left( -\beta_0 e - \beta_1 \dot{e} + a_0 \overline{y} \right) \tag{8}$$

yields to the velocity regulation error dynamics

$$\ddot{e} + (a_1 + \beta_1)\dot{e} + (a_0 + \beta_0)e = 0,$$
(9)

where  $e = y - \overline{y}$  and  $\overline{y}$  denotes the desired operating velocity. Hence, by selecting  $\beta_0 > -a_0$  and  $\beta_1 > -a_1$ , asymptotic velocity regulation is guaranteed:

$$\lim_{t \to \infty} e = 0 \implies \lim_{t \to \infty} y = \overline{y}$$
(10)

Indeed, one can set the control gains as  $\beta_0, \beta_1 > 0$  to improve the controlled velocity response. For instance, the control gains can be chosen to have the following closed-loop error dynamics:

$$\ddot{e} + 2\zeta_c \omega_{nc} \dot{e} + \omega_{nc}^2 e = 0 \tag{11}$$

where  $\omega_{nc}$  and  $\zeta_{c}$  are the natural frequency and damping ratio for the controlled system, respectively.

On the other hand, for synthesis of the proposed dynamic control scheme for both velocity regulation and active suppression of harmonic load torque, a virtual (nonphysical) vibration absorber is considered to be connected to the motor rotor dynamics. Thus, the following velocity control scheme with an embedded virtual vibration absorber is proposed as

$$u = \frac{1}{b} \left[ -\beta_0 e - \beta_1 \dot{e} + a_0 \overline{y} + k_a \left( x_1 - e \right) \right], \tag{12}$$

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$$v = -\beta_{0}e - \beta_{1}\dot{e} + k_{a}(x_{1} - e)$$

$$\dot{x}_{1} = x_{2} , \qquad (13)$$

$$\dot{x}_{2} = -\frac{k_{a}}{J_{a}}(x_{1} - e)$$

where  $x_1$  and  $x_2$  are states of the secondary subsystem coupled virtually to the primary subsystem (motor dynamics). The interested reader on synthesis of dynamic vibration absorbers for active harmonic vibration suppression is referred to the contributions [13, 14].



Figure 1. a) Velocity regulation using the differential flatness-based controller without exogenous harmonic torque ( $\tau_L \equiv 0$ ). b) Electric current signal response. c) Control voltage for velocity regulation for unperturbed operational conditions.

#### 4. Simulation Results

Capability of the proposed dynamic control scheme for both velocity regulation and active suppression of harmonic forced vibrations affecting the rotor dynamics was verified by computer simulations on a DC motor with parameters described in Table 1.



**Figure 2.** Closed-loop system response using the differential flatness-based controller in presence of exogenous harmonic torque: a) Unsatisfactory velocity regulation. b) Electric current signal. c) Control voltage for harmonically perturbed operational conditions.

First of all, control performance for regulating the DC motor rotor at the specified operating velocity  $\overline{y} = \overline{\omega} = 2$  rad/s was confirmed for the operational scenario without problems of forced vibrations ( $F \equiv 0$ ). The effectiveness of the differential flatness controller is confirmed in Fig. 1. Nevertheless, when rotor velocity dynamics is disturbing by harmonic torque, active vibration suppression is not guaranteed by the controller as depicted in Fig. 2.

$R = 2.5 \Omega$	$k_e = 82:3215 \text{ mV/rad/s}$
<i>L</i> = 0:612 H	$J_e = 2:4 \text{ Kgm}2$
$k_m = 82:2 \text{ mNm/A}$	n = 81

 Table 1. DC motor parameters



**Figure 3.** Closed-loop system variables using differential flatness-based control with a virtual vibration absorber: a) Acceptable velocity regulation. b) Electric current signal. c) Control voltage with vibration absorption capability.

On the other side, active vibration suppression and simultaneous acceptable velocity regulation around the specified speed value by implementing the differential flatness control scheme with an embedded virtual dynamic vibration absorber is proved in Fig. 3. Here, the harmonically oscillating torque is described by

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 $\tau_I = \overline{\tau}_I \sin(\Omega t)$ ,

with  $\overline{\tau}_L = 5$  Nm,  $\Omega = 2\pi f$  rad/s, f = 60 Hz. Thus, the virtual vibration absorber was tuned at the frequency  $\Omega$ , with  $J_a = J_e/2$ . Control gains were selected as described by Eq. (12), with  $\omega_{nc} = 15$  rad/s and  $\zeta_c = 0.7071$ .

#### 1. Conclusions

A dynamic control approach for efficient and robust regulation of velocity and active vibration suppression for DC motors was introduced. The active vibration control scheme is based on differential flatness, and virtual dynamic vibration absorbers tuned to suppress harmonic load torque disturbing the motor rotor dynamics. Preliminary simulation results have confirmed that the use of vibration absorbers virtually coupled to the controlled velocity dynamics represents a very choice for harmonic vibration suppression as well as velocity regulation at the same time. Future research works will deal with the implementation of virtual dynamic vibration absorbers to suppress harmonic oscillations in electric power systems.

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